DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

100. Proposed by CHAS. C. CROSS, Libertytown, Md.

I bought stock at 4% discount, and sold it at $2\frac{1}{2}$% premium, paying a brokerage in both cases of $\frac{1}{4}$%. If my net profits were $130, what was my investment ? (Solve by Arithmetic).

I. Solution by W. F. BRADBURY, A.M., Head Master Cambridge Latin School, Cambridge, Mass., and M. E. GRABER, Tiffin, Ohio.

$4\% + 2\frac{1}{2}\% - \frac{1}{2}\%$ (brokerage)$=6\%$.

That is, he made $6 on every $100 he invested. $130 \div 6 = 21\frac{2}{3}$.

He bought $21\frac{2}{3}$ shares of $100 each, or $\$2166\frac{2}{3}$ worth of stock.

But it cost him $3\frac{3}{4}\%$ below par.

$\$2166\frac{2}{3} \times 0.96\frac{1}{4} = \$2085.415$.

Solved in a similar manner by *P. S. BERG* and *W. H. DRANE*.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

"In both cases" is a dubious expression. If the brokerage in the two transactions was *together* $\frac{1}{4}$%, then the net gain on a dollar$=.04+.02\frac{1}{2}-.00\frac{1}{4}=.06\frac{1}{4}$, and the investment$=\$130 \div .06\frac{1}{4} = \$2080$.

But if the brokerage in *each* transaction was $\frac{1}{4}$%, then the net gain on a dollar$=.04+.02\frac{1}{2}-.00\frac{1}{2}=.06$; and the investment$=\$130 \div .06 = \$2166.66\frac{2}{3}$.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa., and ALOIS F. KOVARIK, Instructor in Mathematics, Decorah Institute, Decorah, Iowa.

Let $100=1 share.

$4\% - \frac{1}{4}\% = 3\frac{3}{4}\%$; $100\% + 2\frac{1}{2}\% - \frac{1}{4}\% = 102\frac{1}{4}\%$. $100\% - 3\frac{3}{4}\% = 96\frac{1}{4}\%$.

$\$100 \div .96\frac{1}{4} = \$103.896$.

$\$103.896 \times 1.02\frac{1}{4} = \$106.23366$.

$\$106.23366 - \$100 = \$6.23366$, gain on one share.

$\$130 \div \$6.23366 = 20.85452$ shares.

$20.85452 \times \$100 = \$2085.452$, amount invested.

101. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

A man gained $m=3\%$ on his money, in July; and, in August, lost $n=2\%$. What per cent. of his money July 1st is his money September 1st ?

I. Solution by P. S. BERG, Principal of Schools, Larimore, N. D., and JOHN F. TRAVIS, Student in Ohio State University, Columbus, Ohio.

If 100% is his money on July 1st, then 103% is his money August 1st, and 98% of 103% or 100.94% is his money on September 1st.

Therefore his money September 1st is 100.94% of his money July 1st.

II. **Solution by J. M. COLAW, A. M., Monterey, Va.; W. F. BRADBURY, A. M., Head Master Cambridge Latin School, Cambridge, Mass.; M. A. GRUBER, A. M., War Department, Washington, D. C.; WALTER HUGH DRANE, Graduate Student, Harvard University; and J. O. MAHONEY, B. E., M. Sc., Master of Mathematics and Science, Carthage Graded and High School, Carthage, Texas.**

If p is his principal on July 1st, then $\frac{p(100+m)}{100}$=his money on Aug. 1st.

Also, $\frac{p(100+m)}{100}-\frac{pn(100+m)}{10000}$, or $\frac{p(100-n)(100+m)}{10000}$=his money on September 1st.

$$\therefore \frac{p(100-n)(100+m)}{10000}\div p=\frac{(100-n)(100+m)}{10000},=\left[\frac{(100-n)(100+m)}{100}\right]\%.$$

Substituting numerical values for m and n, respectively, we get $\frac{98\times 103}{10000}=1.0094,=100.94\%$.

The result is independent of the value of p.

Also solved by *G. B. M. ZERR.*

ALGEBRA.

88. **Proposed by E. S. LOOMIS, Ph. D., Professor of Mathematics in Cleveland West High School, Berea, O.**

(1) The Indemnity Savings and Loan Company made two loans of \$1000 each to "A", one of its borrowers, under the following terms: In the first loan "A" agrees to cancel the \$1000 by making 120 payments of \$13.50, the first payment to be considered as made on the first of the month in which the loan is made, and the 119 subsequent payments to be made on the first of each subsequent month; in the second loan "A" agrees to cancel the \$1000 by making 120 payments of \$13.50, the first payment being made on the first of the month following the loan, and the 119 subsequent payments being made on first of the subsequent months. Does the Company sustain any loss in earnings by the second loan over the first loan, and if so how much, and when is (or are) this loss (or these losses) sustained, the rate of interest in each case being considered as 10½% per annum ?

(2) Deduce a formula for each case of proposition (1) by means of which one can find the balance of the loan uncancelled at the end of *any* month, *if* the loan is fully cancelled in 120 payments.

I. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

(1) Let P=principal, r=rate of interest, n=number of payments, x=each payment. Then it has been shown in several previous problems that

$$x=\frac{Pr(1+r)^n}{(1+r)^n-1}.$$

In the first case "A" gets at once \$1000.00−\$13,50=\$986.50, and pays this amount in 119 monthly installments.

$\therefore P=986.50,\ r=10\tfrac{1}{2}\div 12=0.875\%,\ n=119.$

$$\therefore x=\frac{986.50\times .00875(1.00875)^{119}}{(1.00875)^{119}-1}.$$

x=\$13.375 or \$0.125 less than \$13.50.

In the second case $P=1000,\ r=.00875,\ n=120.$